LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

B.Sc. DEGREE EXAMINATION – **PHYSICS**

THIRD SEMESTER – **APRIL 2023**

UPH 3502 – MATHEMATICAL PHYSICS - II

Date: 04-05-2023 Dept. No. ☐ Max. : 100 Marks
Time: 01:00 PM - 04:00 PM

SECTION A

Q. No.	Answer ALL questions		
1	**MCQ** **(5 x 1 = 5)**		
(a)	Which of the following equation is Laplace's equation? (a) $\nabla^2 u = 0$ (b) $\nabla^2 u = \frac{-\rho}{\varepsilon_0}$ (c) $\nabla^2 u = 1$ (d) $\nabla^2 u = -1$	K1	CO1
(b)	In heat flow equation is $\frac{\partial^2 u}{\partial x^2} = \frac{1}{h^2}\frac{\partial u}{\partial t}$, the quantity 'h' is called (a) Planck's constant (b) Conductivity (c) heat flow constant (d) diffusivity	K1	CO1
(c)	When solving a 1-Dimensional wave equation using variable separable method, we get the solution if the constant (P) is (a) positive (b) negative (c) zero (d) one	K1	CO1
(d)	Fourier transform of $F[af_1(x) + bf_2(x)]$ (a) $aF_1(x) + bF_2(x)$ (b) $af_1(x) + bf_2(x)$ (c) $aF_1(s) + bF_2(s)$ (d) $aF(x) + bF(x)$	K1	CO1
(e)	Which of the following is a method of finding roots of an algebraic equation? (a) Newton-Rapshon (b) Lagrange's method (c) Trapezoidal rule (d) Simpson 1/3rd rule.	K1	CO1
2	**Fill in the blanks** **(5 x 1 = 5)**		
(a)	Laplace equation in cylindrical coordinate system is ………………	K1	CO1
(b)	If the roots α and β of second order differential equation are real and distinct, then the general solution is …………………	K1	CO1
(c)	Fourier sine transform of $\frac{1}{x}$ is…………………	K1	CO1
(d)	The Fourier cosine transform of f(x) is………..	K1	CO1
(e)	Trapezoidal rule is used to evaluate ………………… integrals.	K1	CO1
3	**Match the following** **(5 x 1 = 5)**		
(a)	Partial derivative First order differential equation.	K2	CO1
(b)	$F_s\,[e^{-ax}]$ Numerical Integration.	K2	CO1
(c)	Simpson's 1/3rd rule $\sqrt{\frac{2}{\pi}}\frac{s}{a^2+b^2}$.	K2	CO1
(d)	Euler's method $\sqrt{\frac{2}{\pi}}\frac{a}{a^2+b^2}$.	K2	CO1
(e)	$F_c[e^{-ax}]$ Wave equation.	K2	CO1
4	**State True or False** **(5 x 1 = 5)**		
(a)	A partial differential equation is a differential equation in which the dependent variable depends on two or more independent variable.	K2	CO1
(b)	Fourier sine and cosine transforms are used to solve first and second order	K2	CO1

	differential equations.		
(c)	Lagrange's interpolation formula used is only equal intervals.	K2	CO1
(d)	Numerical integration is the technique of computing the value of an indefinite integral.	K2	CO1
(e)	Interpolation is a technique of computing the value of the function within the range of values, of the given parameter.	K2	CO1

SECTION B

Answer any TWO of the following **(2 x 10 = 20)**

5.	Solve the wave equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ using D'Alembert's principle.	K3	CO2
6.	Show that $(i). F_s[xf(x)] = -\frac{d}{ds}F_C(s)$ $(ii)\ F_c[xf(x)] = \frac{d}{ds}F_S(s)$. Hence find Fourier cosine and sine transform of xe^{-ax}.	K3	CO2
7.	Apply least square method to fit a straight line to the data given below. Also estimate the value of y at x =2.5.	K3	CO2

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

8.	Calculate $\int_{-3}^{3} x^4\, dx$ by using Trapezoidal rule and Simpson's one third rule. Verify your answer with actual integration.	K3	CO2

SECTION C

Answer any TWO of the following **(2 x 10 = 20)**

9.	A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form y = k (lx $-x^2$) from which it is released at time t = 0. Find the displacement of any point on the string at a distance of 'x' from one end at time 't'.	K4	CO3
10.	Analyse the one dimensional heat equation and solve it to get its general solution.	K4	CO3
11	Apply Lagrange's formula to calculate f (2.4) from the following table	K4	CO3

x	0	1	2	3
f(x)	1	2	1	10

12	Identify the root of the function f(x) = $x^4 + x^3 - 7x^2 - x + 5 = 0$ correct to three decimal places using Newton-Raphson method.	K4	CO3

SECTION D

Answer any ONE of the following **(1 x 20 = 20)**

13	(i) Solve $\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} = 0$ by the method of separation of variables. (14) (ii) Using the method of separation of variable, solve $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u,\ where\ u(x,0) = 6e^{-3x}$ (6)	K5	CO4
14	Assess the numerical solution of $\frac{dy}{dx} = x + y$, from x=0 to 0.2 by Euler's method with the initial condition's $x_o = 0$, $y_o = 1$ by taking h = 0.025.	K5	CO4

SECTION E

Answer any ONE of the following		**(1 x 20 = 20)**	
15	(i) Find the Fourier transform of the function $$f(x) = \begin{cases} 1 + \frac{x}{a}, & -a < x < 0 \\ 1 - \frac{x}{a}, & 0 < x < a \\ 0 & Otherwise \end{cases}$$ (12) (ii) State and prove convolution theorem in Fourier Transform. (8)	K6	CO5
16	Prepare the forward difference/ backward difference table and estimate the population in the year 1946 of the given data by using Newton's forward and backward interpolation formulas. Also compare the final results.	K6	CO5

Year	1911	1921	1931	1941	1951	1961
Population in thousands	12	15	20	27	39	52

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